Solution to Problems ♠–8

Problem A: *Draw the graph of the function*

$$y = x^{x^{x^{x^{\cdot^{\cdot}}}}}$$

for $0 < x < 1$. *(Use Maple or any other software)*

Answer: Let

$$y = f(x) = x^{x^{x^{x^{\cdot^{\cdot}}}}}$$

If $f(x)$ exists then it satisfies the equation $y = x^y$ or $\ln y = y \ln x$, and thus, we have $x = y^{\frac{1}{y}}$. The inverse function of $f(x)$ is the function $f^{-1}(x) = x^{\frac{1}{x}}$. We can sketch the curve $y = x^{\frac{1}{x}}$ on the interval $(0, 1)$ and then we reflect the graph about the line $y = x$ to get the graph of $f(x)$.

Correct solutions were received from :

(1) Brad Tuttle POW 8A: ♠

Problem B: *Let n and m be integer numbers. Prove that, if $3n+7m$ is divisible by 19, then the number $43n+75m$ is divisible by 19 also.*

Answer: Let $3n+7m$ be divisible by 19. Then $43n+75m = 8(3n+7m)+19(n+m)$ and it is also divisible by 19.

Correct solutions were received from :

(1) Brad Tuttle POW 8B: ♠